Woodvale Secondary College

WA Exams Practice Paper B, 2016

Question/Answer Booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures

In words ________________________________

Your name ________________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	11	11	100	98	65
			Total	149	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 (6 marks)

The number of telephone calls each day to a particular number during weekdays are shown in the table below.

	Daily calls					
Week	Mon	Tue	Wed	Thu	Fri	Weekly mean
1	21	18	26	22	18	A
2	21	17	24	21	17	20
3	22	19	27	23	B	22

(a) Calculate the values of A and B in the table. (2 marks)

$$A = \frac{21+18+26+22+18}{5} = 21$$

$$\frac{22+19+27+23+B}{5} = 22 \Rightarrow B = 19$$

(b) Determine the seasonal index for Wednesday, correct to two decimal places. (3 marks)

$$\frac{26}{21}+\frac{24}{20}+\frac{27}{22} = 1.238+1.2+1.227$$

$$= 3.665$$

$$3.665 \div 3 = 1.221$$

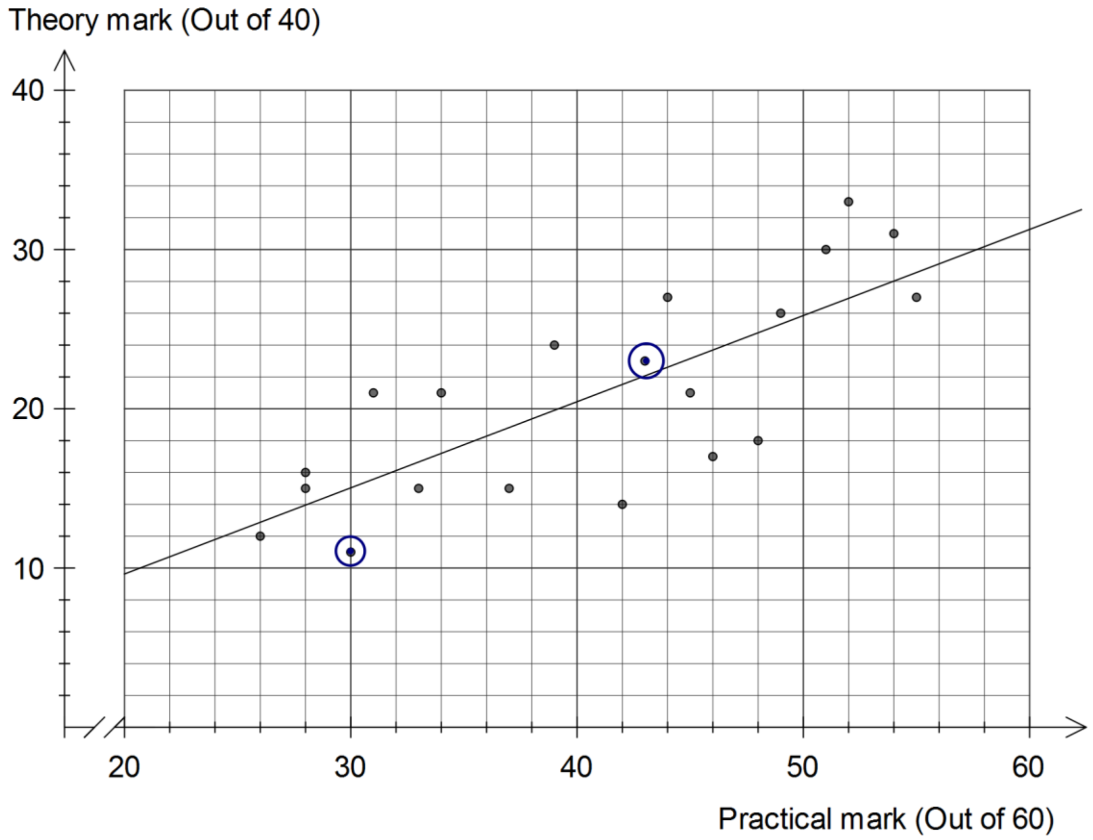
$$\approx 1.22$$

(c) Deseasonalise the number of calls on Wednesday of Week 3, giving your answer correct to one decimal place. (1 mark)

$27 \div 1.22 = 22.1$ (1dp)

Question 9 **(12 marks)**

To be awarded a certificate of competence at the end of a photography course, participants sat a theory test out of 60 marks and a practical test out of 40 marks. The results of 18 students are shown on the graph below.



(a) Two other students sat the tests, one scoring 43 in the practical and 23 in the theory, and the other scoring 30 in the practical and 11 in the theory. Add these points to the graph. (2 marks)

(b) State the highest score recorded for the theory test. (1 mark)

33

(c) How many students scored a mark greater than 45 in the practical test? (1 mark)

7 students

The equation of the least squares line for the 20 data points is $y = 0.54x - 1.05$, where x is the theory mark and y is the practical mark. The correlation coefficient is 0.82.

(d) Draw the least squares line on the graph. (2 marks)

(e) Calculate the coefficient of determination and interpret the value of the coefficient in this context. (2 marks)

$R^2 = 0.82^2 = 0.67$ (2dp)

We can conclude that approximately 67% of the variation in theory marks of the students can be explained by the variation in their practical marks.

(f) Predict the theory mark, to the nearest whole number, of a student who scored

(i) 50 in the practical test. (1 mark)

26

(ii) 22 in the practical test. (1 mark)

11

(g) Which, if any, of the predictions made in (e) do you consider to be unreliable? Explain your answer. (2 marks)

The second prediction (using 22), as it involves extrapolation beyond the lowest practical score.

Question 10 **(8 marks)**

The value of an investment n months after an initial amount of \$ A is deposited into a savings bank can be calculated by the recursive rule

$T_n = \left(1 + \frac{R}{100}\right)T_{n-1}, \quad T_0 = A$, where R is the monthly interest rate as a percentage.

(a) An initial amount of \$1000 is deposited with a savings bank offering an interest rate of 0.4% per month.

(i) What is the value of n after one year? (1 mark)

$\boxed{12}$

(ii) What is the value of the investment after one year, to the nearest cent? (2 marks)

$T_{12} = 1049.0702$

Value is \$1049.07

(iii) How much interest has accumulated over the first year? (1 mark)

$\boxed{1049.07 - 1000 = \$49.07}$

(b) Consider the recursive rule $T_n = 1.0025T_{n-1}, \quad T_0 = 3450$, used to calculate the value of another investment after n months.

(i) What is the initial amount of the investment? (1 mark)

$\boxed{\$3\ 450}$

(ii) What is the monthly interest rate as a percentage? (1 mark)

$\boxed{0.0025 \times 100 = 0.25\%}$

(iii) How much interest would accumulate over the first year of this investment? (2 marks)

$T_{12} = 3554.94$

$3554.94 - 3450 = \$104.94$

Question 11 **(8 marks)**

A plant grew from a seed to a height of 120 cm in its first year. The growth of the plant in subsequent years is expected to be 60% of its growth in the previous year.

(a) Determine

(i) the growth of the plant during the second year. (1 mark)

$120 \times .6 = 72$ cm

(ii) the height of the plant after two years. (1 mark)

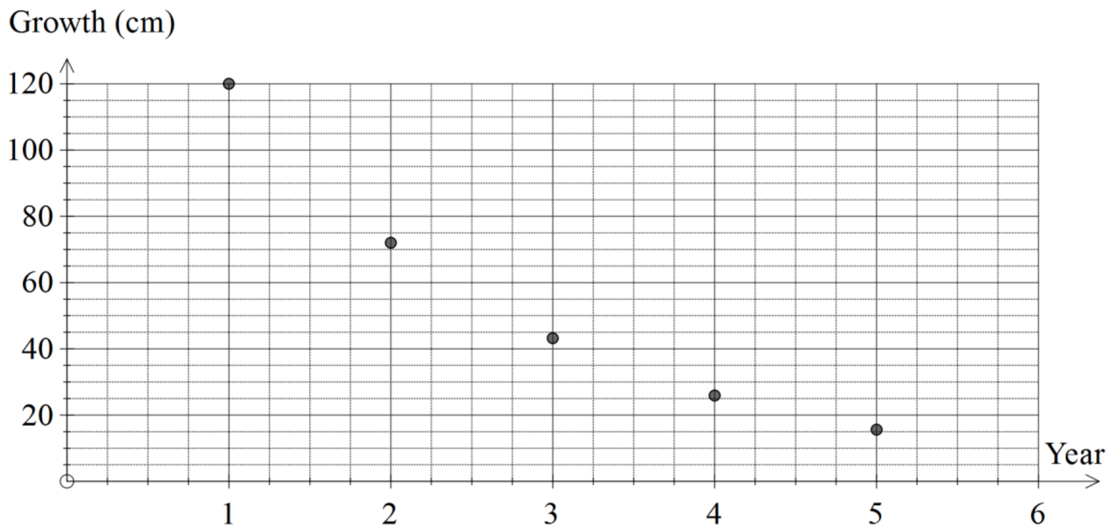
$120 + 72 = 192$ cm

The growth of the plant during the n^{th} year can be given by $T_{n+1} = 0.6T_n$, $T_1 = 120$.

(b) Complete the growth table below. (2 marks)

Year	1	2	3	4	5
Growth (cm)	120	72	43.2	25.9	15.6

(c) Plot the annual growth of the plant on the axes below for the first five years. (2 marks)



(d) In which year is the growth of the tree first less than 1 cm? (1 mark)

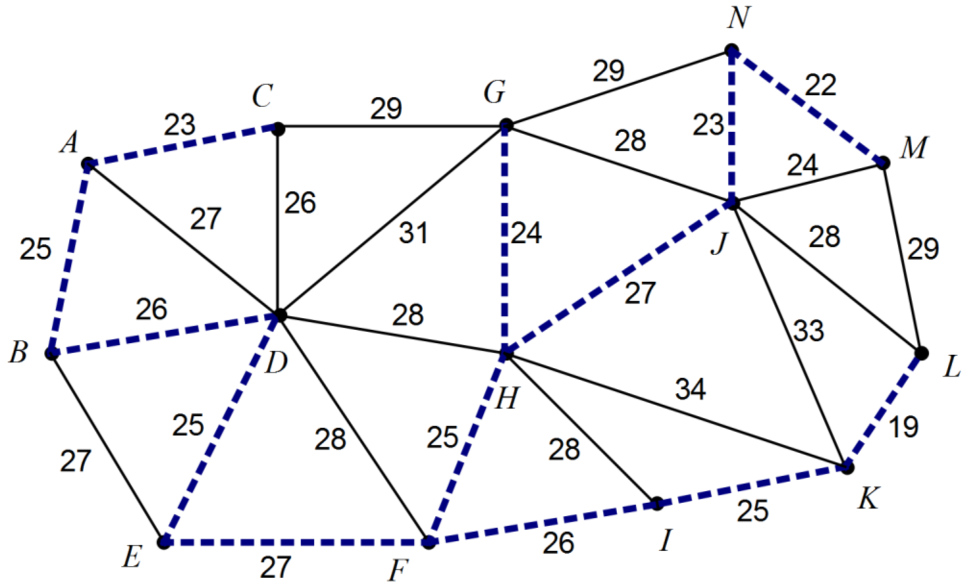
Year 11 (0.7 cm)

(e) Describe height of the tree in the long-term. (1 mark)

The growth of the tree each year is rapidly decreasing as time goes on, and so the height will eventually reach a maximum. (300 cm).

Question 12 **(8 marks)**

An electrical supply has to be made to the 14 buildings on a new industrial site. The cost of establishing a direct electrical supply between various buildings is shown on the graph below, where the number on each edge represents the cost for that link, in thousands of dollars.



(a) Clearly indicate the minimal spanning tree on the graph above. (3 marks)

(b) Determine the minimum cost of establishing the electrical supply to the 14 buildings. (2 marks)

$23+25+26+25+27=126$

$26+25+19=70$

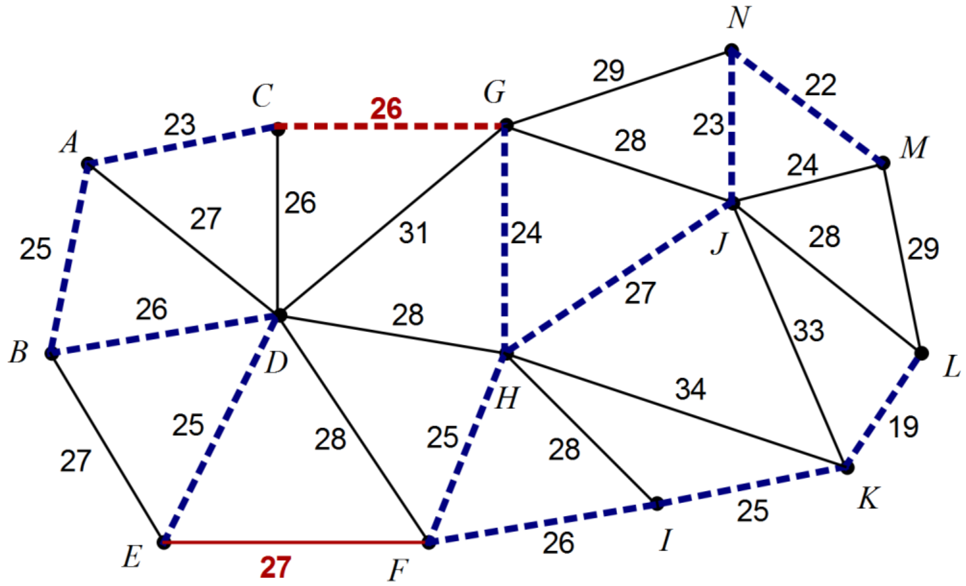
$25+24+27+23+22=121$

Total $=317$

Cost is $317\times1000=\$317\ 000$

(c) If the cost of the link between C and G could be decreased by $3 000, explain what effect, if any, this would have on the solution in part (b). Justify your answer. (3 marks)

A copy of the original graph is provided for your use, if required.



Link CG decreases from 29 to 26.

Replace link EF (27) with CG (26) in minimal spanning tree, saving $1 000.

Minimum cost in (b) has decreased to $316 000.

Question 13 **(11 marks)**

Some of the quarterly profits (in thousands of dollars) for an export business over 16 consecutive quarters, together with some seasonal indices, are shown in the table below.

Year	Quarter	Time (t)	Profit ($000's)	Seasonal indices
2010	1	1	48	1.34
2010	2	2	37	1.03
2010	3	3	26	**A**
2010	4	4	32	**B**
2011	1	5	44	1.34
2011	2	6	**C**	1.07
2011	3	7	22	0.67
2011	4	8	30	0.92
2012	1	9	36	1.40
2012	2	10	25	0.97
2012	3	11	15	0.58
2012	4	12	27	1.05
2013	1	13	33	1.36
2013	2	14	25	1.03
2013	3	15	16	0.66
2013	4	16	23	0.95

(a) Determine the seasonal indices A and B in the table above, rounding your answers to two decimal places. (3 marks)

$$(48+37+26+32)\div 4=35.75$$

$$A=26\div 35.75=0.7272...\approx 0.73$$

$$B=32\div 35.75=0.8951...\approx 0.90$$

(b) Given that the 4-point centred moving average associated with Quarter 4 of 2011 is 30, determine the value of C in the table above.

(2 marks)

$$\frac{\frac{C}{2}+22+30+38+\frac{25}{2}}{4}=30$$

$$C=35$$

The seasonal index for the third quarter is 0.66 and for the fourth quarter is 0.95.

(c) Calculate the seasonal index for the first quarter of the year. (1 mark)

$$(1.34+1.34+1.40+1.36)\div 4=5.44\div 4=1.36$$

(d) Calculate the deseasonalised profit for the fourth quarter of 2012. (2 marks)

$$27 \div 0.95 = 28.421$$

$$28.421 \times 1000 \approx \$28\ 400$$

(e) The equation of the least-squares regression line fitted to the deseasonalised profit figures (p in 000's) against time (t) is $p = 38.36 - 1.0127t$. Forecast the expected profit in Quarter 3, 2014 if trends in the above data continue. (3 marks)

$$t = 19$$

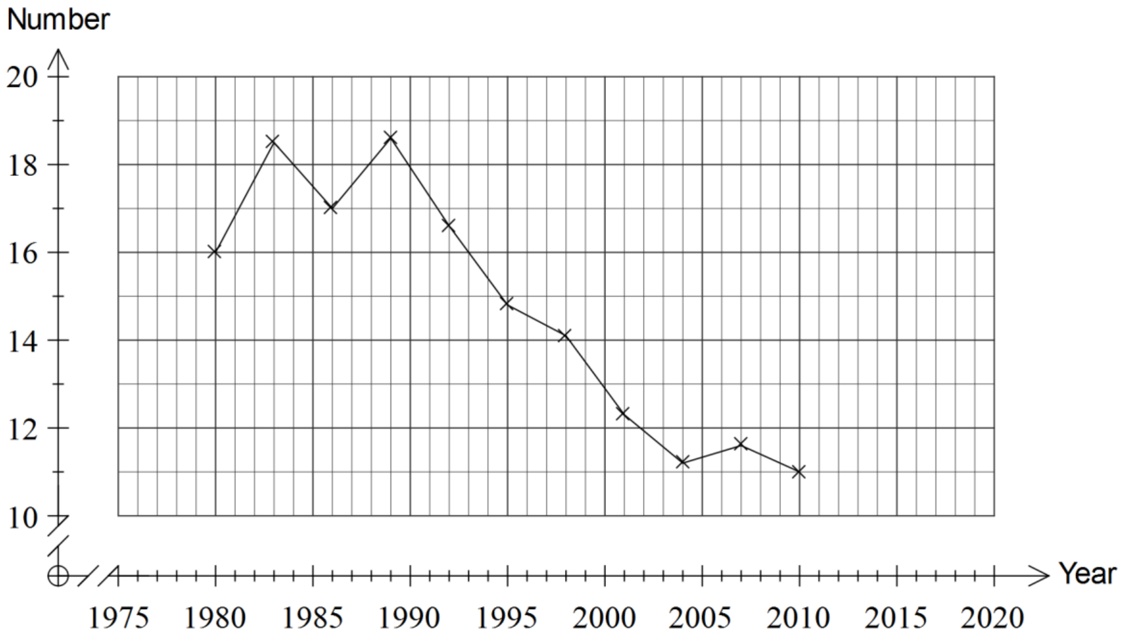
$$\begin{aligned} m &= 38.36 - 1.0127(19) \\ &= 19.12 \end{aligned}$$

$$19.12 \times 0.66 = 12.62$$

$$\begin{aligned} \text{Profit } &= 1000 \times 1262 \\ &\approx \$12\ 620 \end{aligned}$$

Question 14 **(10 marks)**

The graph below shows the average number of cigarettes smoked per week by smokers aged 18 to 24 for the years 1980 to 2010.



The table below shows the same data for 1989 to 2010.

Year, x	1989	1992	1995	1998	2001	2004	2007	2010
Number, y	18.6	16.6	14.8	14.1	12.3	11.2	11.6	11

(a) For the data in the table, determine

(i) the correlation coefficient r_{xy}. (1 mark)

-0.956

(ii) the least squares regression model, giving coefficients rounded to four decimal places. (2 marks)

$y = -0.3603x + 734.2298$

(b) Describe how your answers in (a) would be affected if all the data shown in the graph was used rather than that displayed in the table. (2 marks)

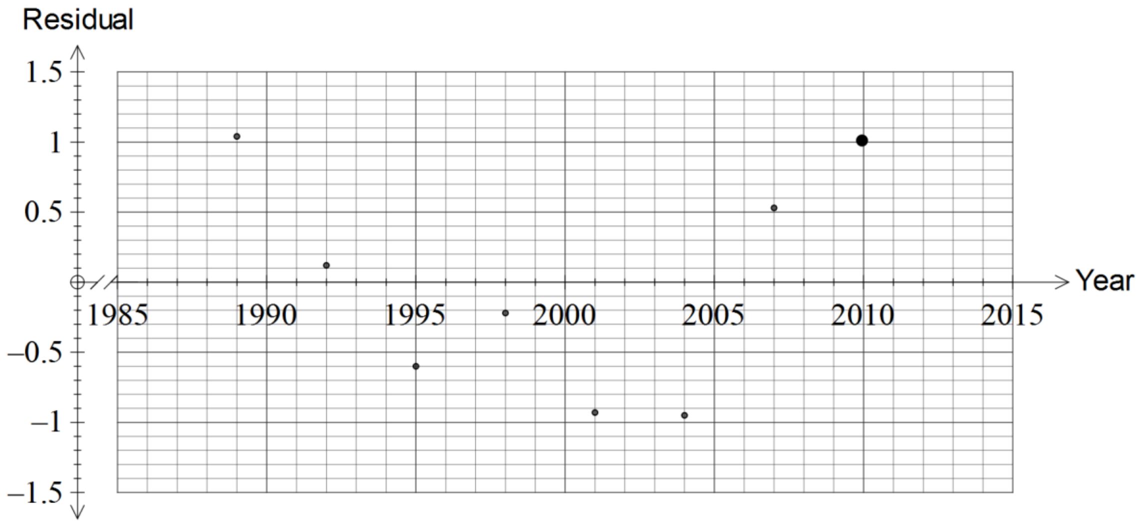
- correlation would weaken and move away from -0.96 towards 0.

- regression line would be flatter (coefficient of -0.36 would move towards 0) and y-intercept would decrease from 734.

(c) Use the regression model from (a) to calculate the residual for 2010 and plot this point on the residual graph below. Show all your working. (3 marks)

$$\hat{y} = -0.3603(2010) + 734.2298$$
$$\approx 10$$

$$y - \hat{y} = 11 - 10$$
$$= 1$$

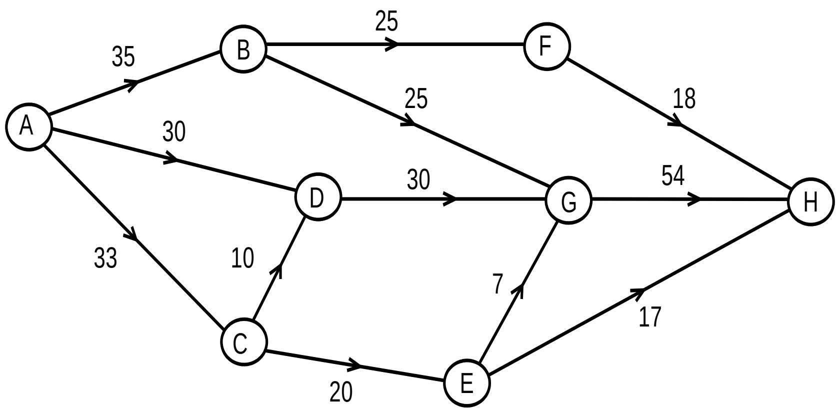


(d) Use the residual plot above to comment on the appropriateness of fitting a linear model to the data in the table. (2 marks)

Fitting linear model is not appropriate as the residuals clearly display a pattern.

Question 15 **(8 marks)**

The maximum numbers of shipping containers that can be moved between various points in a distribution network each day are shown in the weighted digraph below.



(a) State the source and the sink of this digraph. (1 mark)

Source = A

Sink = H

(b) What is the maximum number of containers that can be transported from the source to the sink each day? Show systematic working to allow your solution to be checked. (4 marks)

ABFH = 18
ABGH = 17
ADGH = 30
ACEGH = 7
ACEH = 13

Total = 85 containers per day

(c) One day, a shortage of drivers meant that the number of containers which could be sent from A to D was halved. What effect, if any, would this have on the maximum number of containers that can be transported each day? Justify your answer. (3 marks)

Lose 15 from ADGH.

Add 10 along ACDGH.

Net result is to lose 5, so maximum is now just 80 containers each day.

Question 16 **(8 marks)**

A young person has borrowed $7 500 to purchase a car and is making repayments of $660 at the end of each month on the loan, with interest charged monthly.

The spreadsheet below shows the balance and interest of the loan for the first three months.

Month	Balance at start of month ($)	Interest for month ($)
1	7 500.00	52.50
2	6 892.50	48.25
3	6 280.75	43.97

(a) Use information from the table to show that the annual interest rate on the loan is 8.4% per annum. (2 marks)

$$\frac{52.50}{7500} = 0.007$$

$$0.007 \times 12 \times 100 = 8.4\%$$

(b) Determine the balance at the start of month 4, and the interest for this month. (2 marks)

Bal: $6280.75 + 43.97 - 660.00 = \$5664.72$

Int: $5664.72 \times 0.007 = \$39.65$

(c) Write a recursive rule to determine the balance at the start of each month. (2 marks)

$$T_{n+1} = T_n \times 1.007 - 660, \quad T_1 = 7500$$

(d) The loan is fully repaid by the end of month 12. Determine, to the nearest dollar,

(i) the amount of the 12th (last) repayment. (1 mark)

$$582.68 \approx \$583$$

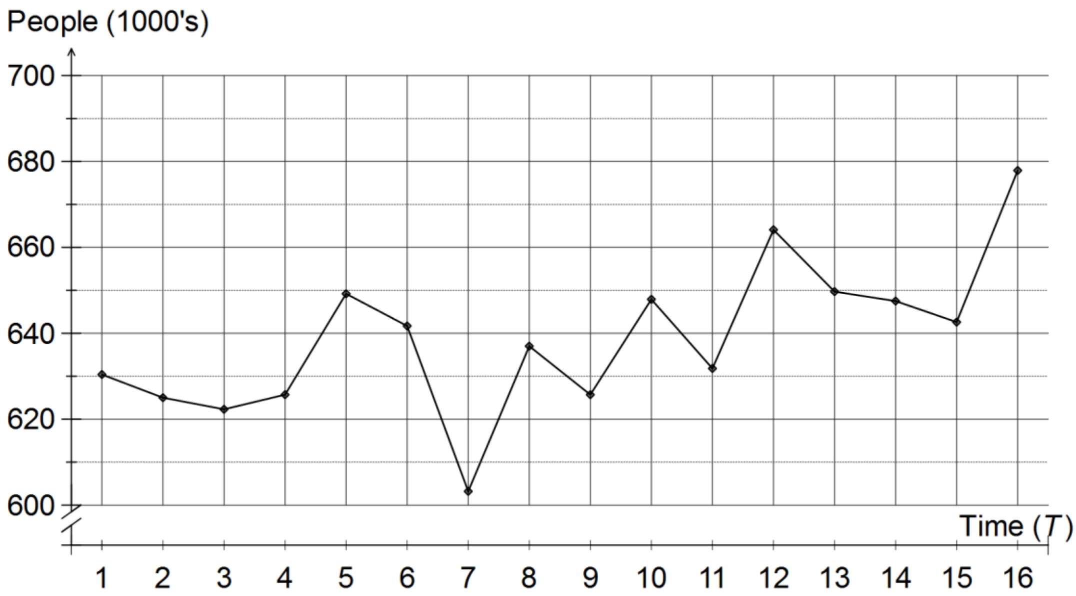
(ii) the total amount of interest paid over the twelve months. (1 mark)

$$11 \times 660 + 583 - 7500 = \$343$$

Question 17 **(10 marks)**

The table and graph below show the number of people employed in the accommodation and food services industry in Australia from February 2001 to November 2004.

Year	Quarter	Time (T)	People (1000's)	Annual index
2001	Feb	1	630.4	1.007
2001	May	2	625.0	0.999
2001	Aug	3	622.3	0.994
2001	Nov	4	625.7	1.000
2002	Feb	5	649.2	1.026
2002	May	6	641.7	1.014
2002	Aug	7	603.2	0.953
2002	Nov	8	637.0	1.007
2003	Feb	9	625.7	0.974
2003	May	10	647.9	1.009
2003	Aug	11	631.8	0.984
2003	Nov	12	664.1	1.034
2004	Feb	13	649.7	0.993
2004	May	14	647.5	0.989
2004	Aug	15	642.6	0.982
2004	Nov	16	677.9	1.036



(a) How do the data points for August support the use of a four-point centred moving average to smooth the entire data set? (1 mark)

The data points for August tend to be the lows for each year, suggesting that there is a cycle of four quarters to the data.

(b) Calculate the four-point centred moving average for August 2001. (2 marks)

$$\frac{\frac{630.4}{2}+625.0+622.3+625.7+\frac{649.2}{2}}{4}=628.2$$

The seasonal indices for three quarters are shown in the table below.

Day	Feb	May	Aug	Nov
Seasonal index	1.000	1.003	0.978	

(c) Determine the seasonal index for November. (1 mark)

$$4-(1.000+1.003+0.978)=1.019$$

The following table shows the deseasonalised values of the number of people, P.

	2001				2002				2003				2004			
Qtr	F	M	A	N	F	M	A	N	F	M	A	N	F	M	A	N
T	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
P	630	623	636	614	649	640	617	625	626	646	646	652	650	646	657	665

(d) Show how the deseasonalised value of 657 for P in August 2004 was calculated. (1 mark)

$$642.6 \div 0.978 = 657.06$$

(e) Use the table of deseasonalised values to determine the equation of the least-squares regression line that can be used to predict P from time T . (2 marks)

$$P = 2.265T + 619.625$$

(f) Forecast the expected number of people employed in the accommodation and food services industry in Australia in August 2006 to the nearest 1 000, and comment on the reliability of your forecast. (3 marks)

$$\begin{aligned} P &= 2.265(23) + 619.625 \\ &= 671.7 \end{aligned}$$

$$\begin{aligned} 671.7 \times 0.978 &= 656.9 \\ &\approx 657\ 000 \text{ people} \end{aligned}$$

This forecast should be treated with caution as it involves considerable extrapolation beyond the given dataset.

Question 18 **(9 marks)**

An annuity paying a monthly sum of \$1 500 is set up with an initial sum of \$400 000 and interest of 5.4% per annum compounded monthly.

(a) The balance of the loan at the start of month n is given by the recurrence relation $A_{n+1} = rA_n - d,\ A_1 = 400000$. State the values of r and d. (2 marks)

$0.054 \div 12 = 0.0045 \Rightarrow r = 1.0045$

$d = 1500$

(b) Determine the value of the annuity after twelve months and comment on what this figure indicates. (2 marks)

\$403 690.45

The value has increased, indicating that the annuity can pay out \$1 500 for ever and the value of the annuity will also increase for ever.

Compound Interest

N	12
I%	5.4
PV	−400000
PMT	1500
FV	403690.4501
P/Y	12
C/Y	12

(c) Determine, to the nearest dollar, the monthly sum that should be withdrawn from the annuity if

(i) the annuity is to last for 20 years. (1 mark)

\$2729

Compound Interest

N	240
I%	5.4
PV	−400000
PMT	2729.006361
FV	0
P/Y	12
C/Y	12

(ii) the annuity is to be a perpetuity. (1 mark)

\$1800

(d) If the interest rate of 5.4% was halved after one year, calculate the total interest accrued by the annuity over the first two years. (3 marks)

During first year: $403690.45 - 400000 + 12 \times 1500 = 21690.45$.

During second year: $396501.56 - 403690.45 + 12 \times 1500 = 10811.11$.

Total: $21690.45 + 10811.11 = \$32501.56$

End of questions

Additional working space

Question number: _________